EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

Prove that :

1. $R\,r\,(\sin A + \sin B + \sin C) = \Delta$

2. $2R\cos A = 2R + r - r_1$

3. $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \frac{s^2}{\Delta}$

4. $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + \frac{r}{2R}$

5. $\frac{\tan\frac{A}{2}}{(a-b)(a-c)} + \frac{\tan\frac{B}{2}}{(b-a)(b-c)} + \frac{\tan\frac{C}{2}}{(c-a)(c-b)} = \frac{1}{\Delta}$

6. If $r_1 = r + r_2 + r_3$ then prove that the triangle is a right angled triangle.

7. If two times the square of the diameter of the circumcircle of a triangle is equal to the sum of the squares of its sides then prove that the triangle is right angled.

8. In acute angled triangle ABC, a semicircle with radius r_a is constructed with its base on BC and tangent to the other two sides r_b and r_c are defined similarly. If r is the raidus of the incircle of triangle ABC then prove that, $\frac{2}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c}$.

9. For any triangle ABC , if B = 3 C , show that

$\cos C = \sqrt{\frac{b+c}{4c}}$ & $\sin\frac{A}{2} = \frac{b-c}{2c}$.

10. In a triangle ABC, BD is median. If $\ell(BD) = \frac{\sqrt{3}}{4}.\ell(AB)$ and $\angle DBC = \frac{\pi}{2}$. Determine the $\angle ABC$.

11. ABCD is a trapezium sum that AB, DC are parallel & BC is perpendicular to them. If angle ADB = θ, BC = p & CD = q, show that $AB = \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$.

12. Find the angles of a triangle in which the altitude and a median drawn from the same vertex divide the angle at that vertex into 3 equal parts.

13. ABCD is a rhombus. The circumradii of ΔABD and ΔACD are 12.5 and 25 respectively. Find the area of rhombus.

14. In a triangle ABC if $a^2 + b^2 = 101c^2$ then find the value of $\frac{\cot C}{\cot A + \cot B}$.

15. If I be the in-centre of the triangle ABC and x, y, z be the circmradii of the triangle IBC, ICA & IAB, show that $4R^3 - R(x^2 + y^2 + z^2) - xyz = 0$.

16. If in a triangle ABC, $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, prove that the triangle ABC is either isosceles or right angled.

17. In a Δ ABC,

(i) $\frac{a}{\cos A} = \frac{b}{\cos B}$

(ii) $2\sin A\cos B = \sin C$

(iii) $\tan^2\frac{A}{2} + 2\tan\frac{A}{2}\tan\frac{C}{2} - 1 = 0$,

prove that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (i).

18. If p_1, p_2, p_3 are the altitudes of a triangle from the vertices A, B, C & Δ denotes the area of the triangle, prove that $\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{2ab}{(a+b+c)\Delta}\cos^2\frac{C}{2}$.

19. The triangle ABC (with side lengths a, b, c as usual) satisfies $\log a^2 = \log b^2 + \log c^2 - \log(2bc\cos A)$. What can you say about this triangle ?

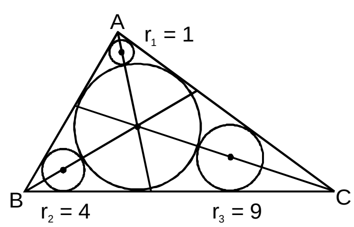
20. If the bisector of angle C of triangle ABC meets AB in D & the circumcircle in E prove that,

$$\frac{CE}{DE} = \frac{(a+b)^2}{c^2}.$$

21. With reference to a given circle, A_1 and B_1 are the areas of the inscribed and circumscribed regular polygons of n sides, A_2 and B_2 are corresponding quantities for regular polygons of 2n sides. Prove that
(1) A_2 is a geometric mean between A_1 and B_1.
(2) B_2 is a harmonic mean between A_2 and B_1.

22. The triangle ABC is a right angled triangle, right angle at A. The ratio of the radius of the circle circumscribed to the radius of the circle escribed to the hypotenuse is, $\sqrt{2}$: ($\sqrt{3}$ + $\sqrt{2}$). Find the acute angle B & C. Also find the ratio of the sides of the triangle other than the hypotenuse.

23. ABC is a triangle. Circles with radii as show are drawn inside the triangle each touching two sides and the incircle. Find the radius of the incircle of the ΔABC.



24. In a scalene triangle ABC the altitudes AD & CF are dropped from the vertices A, C to the sides BC & AB. The area of Δ ABC is know the be equal to 18, the area of triangle BDF is equal to 2 and length of segment DF is equal to $2\sqrt{2}$. Find the radius of the circle circumscribed.